**Directional (One-Tailed) Hypothesis Tests**

- In a directional hypothesis test, or a one-tailed test, the statistical hypotheses (H0 and H1) specify either an increase or a decrease in the population mean. That is, they make a statement about the direction of the effect

**Scenario:**

Imagine a company sells "FastGrow" plant fertilizer. They claim that using their fertilizer will make tomato plants grow *taller* than the average height of 50 centimeters. A gardener wants to test if this claim is true.

**Hypotheses:**

* **Null Hypothesis (H0):** FastGrow fertilizer does *not* make tomato plants grow taller than 50 cm. (The average height is 50 cm or less).
* **Alternative Hypothesis (H1):** FastGrow fertilizer *does* make tomato plants grow taller than 50 cm. (The average height is more than 50 cm).

**Why One-Tailed?**

* The gardener is only interested in whether FastGrow makes plants *taller*. They don't care if it makes them shorter. This is why we use a one-tailed test (specifically, a right-tailed test).

**The Experiment:**

1. **Planting:** The gardener plants 20 tomato seedlings and uses FastGrow fertilizer on all of them.
2. **Measuring:** After a set period, they measure the height of each tomato plant.
3. **Calculating:** They calculate the average height of the 20 plants. Let's say the average height is 53 cm. They also calculate the standard deviation of the height of the 20 plants.

**The Test:**

* We'll use a t-test because we're dealing with a sample and we don't know the population standard deviation.
* We calculate the t-statistic. This tells us how far our sample mean (53 cm) is from the claimed mean (50 cm), taking into account the variability of our data.
* We need to define a significance level (alpha). Let's say we choose 0.05. This means we're willing to accept a 5% chance of being wrong.
* We determine the degrees of freedom. This is the sample size minus one. In this case, 20-1=19.
* We use a t-table or calculator to find the critical t-value for a one-tailed test with 19 degrees of freedom and an alpha of 0.05. This critical value is the threshold.
* If the calculated t-statistic is *greater* than the critical t-value, we reject the null hypothesis.

**Decision:**

* If our calculated t-statistic is greater than the critical t-value, it means our sample average is far enough above 50 cm to be statistically significant.
* In simple words, it's unlikely we'd get a sample average of 53 cm just by chance if FastGrow didn't actually make plants taller.
* Therefore, we conclude that FastGrow fertilizer *does* seem to make tomato plants grow taller.
* If the calculated t-statistic is less than the critical t-value, then we fail to reject the null hypothesis.

**Key Takeaways:**

* **One-tailed:** We're only looking for an effect in one direction (taller).
* **Critical Value:** This is the line that separates "likely" from "unlikely" results.
* **T-Statistic:** This tells us how far our sample result is from the claimed result.
* **Significance Level (alpha):** This is our tolerance for being wrong.
* **Degrees of Freedom:** This accounts for the sample size.

**In a Nutshell:**

We're asking, "Is the average height of our plants so much higher than 50 cm that it's unlikely to happen by chance?" If the answer is yes, we can say FastGrow works.

**Comparision between One-tailed and Two-tailed test**

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| **Characteristics** | **One-tailed test** | **Two-tailed test** |
| Purpose | To test for a difference in a specific direction (increase or decrease) | To test for a difference in any direction (difference) |
| Null hypothesis (H0) | μ ≤ value (right-tailed) or μ ≥ value (left-tailed) | μ = value |
| Alternative hypothesis (H1) | μ > value (right-tailed) or μ < value (left-tailed) | μ ≠ value |
| Critical Region | Located entirely in one tail of the distribution | Divided equally between both tails of the distribution |
| Critival Value | Less extreme (with the same significance level) | More extreme (with the same significance level) |
| Statistical Power | Higher if the predicted direction is correct, lower if incorrect | Lower than one-tailed (if the direction is correct) |
| Risk | Missing an effect if the predicted direction is wrong | Less likely to miss an effect, but harder to detect small effects |
| When to use | When there's a strong prediction about the direction of the effect | When uncertain about the direction of the effect, or want to test for differences in both directions |
| Example | "Drug A reduces blood pressure" | "Does Drug A affect blood pressure?" |